Name:__________________________

SM3 4.2: Multiplying & Dividing Rationals

Simplify the expression and state any restrictions on x.

1) $\frac{3}{x} \cdot \frac{5x}{6x^2+9}$

$\frac{5}{2x^2+3}$; $x \neq 0$

2) $\frac{x}{x-3} \cdot \frac{2x+7}{x+1}$

$\frac{2x^2+7x}{(x-3)(x+1)}$; $x \neq \{3, -1\}$

3) $\frac{x^2+3x}{x-4} \cdot \frac{x^2+1}{x^2}$

$\frac{x^3+3x^2+x+3}{x(x-4)}$, $x \neq \{0, 4\}$

4) $\frac{x}{12} \cdot \frac{8+4x}{5x}$

$\frac{2+x}{15}$; $x \neq 0$

5) $\frac{2x+6}{x-6} \cdot \frac{x^2-4x-12}{30+4x-2x^2}$

$\frac{-x-2}{x-5}$; $x \neq \{6, 5, -3\}$

6) For what values of x is $\frac{2x-8}{x^2+7x+10}$ an invalid expression?

Denominator: $(x+5)(x+2)$ so $x \neq \{-5, -2\}$

7) Show that the rational expression $\frac{5x+5}{x} \cdot \frac{x^3+3x^2}{x^2-1} \cdot \frac{x-1}{5x}$ is equivalent to the rational expression $x+3$. State any restrictions on x.

$\frac{5(x+1)}{x} \cdot \frac{x^2(x+3)}{(x+1)(x-1)} \cdot \frac{x-1}{5x} = x+3;\ x \neq \{0, 1, -1\}$

8) What simplified rational expression represents the area of a rectangle with a width of $\frac{x}{2}$ inches and a length of $\frac{2x+1}{x-5}$ inches? State any restrictions on x.

$A = lw = \frac{x}{2} \cdot \frac{2x+1}{x-5} = \frac{2x^2+x}{2x-10}\ inches^2;\ x \neq 5$

9) The momentum of an object is the product of the object's mass and its velocity. What simplified expression describes the momentum (in $g \cdot cm/s$) of a moving toy car if the rational expression $\frac{x^2+4x+4}{x^2-9}$ describes the toy car's mass in grams, and the expression $\frac{x^2+5x+6}{x+2}$ describes the toy car's velocity in centimeters per second (where $x > 3$)?

$\frac{x^2+4x+4}{x^2-9} \cdot \frac{x^2+5x+6}{x+2} = \frac{(x+2)^2}{x-3}\ g \cdot \frac{cm}{s};\ x > 3$

10) What simplified expression describes the volume of a box with a rectangular base with a length of $\frac{x^2+x}{x-5}$ meters, a width of $\frac{7x}{x+1}$ meters, and height of $\frac{x^2+x-30}{x^2+7x}$? State any restrictions on x.

$$V = lwh = \frac{x^2+x}{x-5} \cdot \frac{7x}{x+1} \cdot \frac{x^2+x-30}{x^2+7x} = \frac{7x^2+42x}{x+7}\ meters^3;\ x \neq \{5, -1, 0, -7\}$$

For problems 11-14, simplify the expression and state any restrictions on x.

11) $\frac{2x}{7} \div \frac{1}{x}$

$\frac{2x^2}{7};\ x \neq 0$

12) $\frac{3}{5x} \div \frac{5}{x}$

$\frac{3}{25};\ x \neq 0$

13) $\frac{8x+3}{5} \div \frac{x}{9}$

$\frac{72x+27}{5x};\ x \neq 0$

14) $\frac{2x-4}{x+1} \div \frac{x}{x+2}$

$\frac{2x^2-8}{x(x+1)};\ x \neq \{0, -2, -1\}$

15) Show that the expression $\frac{x^2-2x-15}{2x^2-8x} \cdot \frac{32-2x^2}{x^2-13x+40} \div \frac{x+4}{8x-x^2}$ is equivalent to the expression $x+3$. State any restrictions on x.

$$\frac{(x-5)(x+3)}{2x(x-4)} \cdot \frac{2(4+x)(4-x)}{(x-5)(x-8)} \div \frac{x+4}{x(8-x)} = \frac{(x-5)(x+3)}{2x(x-4)} \cdot \frac{2(4+x)\cdot -1(x-4)}{(x-5)(x-8)} \cdot \frac{x \cdot -1(x-8)}{x+4} =$$

$$--(x+3) = x+3;\ x \neq \{0, 4, 5, 8, -4\}$$

16) Use synthetic division to rewrite the rational expression $\frac{2x^2-7x-15}{x-5}$ in the form $q(x) + \frac{r(x)}{d(x)}$.

$2x+3$

17) Use polynomial long division to rewrite the rational expression $\frac{x^2-8x-1}{x+1}$ in the form $q(x) + \frac{r(x)}{d(x)}$.

$x - 9 + \frac{8}{x+1}$

18) One way to find an average velocity is to take the quotient of distance traveled and the time required to do so. Find the velocity (in kilometers per hour) of car that covers a distance of $2x + 4$ kilometers in a total time, in hours, described by the expression $\frac{3x-6}{x-2}$ (where $x > 2$).

$$(2x+4) \div \frac{3x-6}{x-2} = \frac{2(x+2)}{3}\ kph, x > 2$$

19) Density is the ratio of an object's mass to its volume. What simplified rational expression represents the density (in grams per cubic meter) of an alloy ingot whose mass, in grams, is described by the expression $\frac{6x^2+x-15}{x+4}$, with a volume in cubic meters represented by the expression $\frac{2x^2-11x+12}{5x}$ (where $x > 4$)?

$$\frac{6x^2+x-15}{x+4} \div \frac{2x^2-11x+12}{5x} = \frac{15x^2+25x}{(x+4)(x-4)} g/m^3, x > 4$$

20) The area of a right triangle, in square meters, is represented by the rational expression $\frac{x^2-3x+5}{x-2}$, where $x > 2$. If the length of one leg, in meters, is described by the rational expression $\frac{x^2-5}{x-1}$, what simplified rational expression represents the other leg?

$$\frac{x^2-3x+5}{x-2} \div \frac{x^2-5}{2(x-1)} = \frac{2(x^2-3x+5)(x-1)}{(x-2)(x^2-5)} = \frac{2x^3-8x^2+16x-10}{(x-2)(x^2-5)}\ m,$$
$x > 2$ and $x \neq \sqrt{5}$